Calculation of the noninertial space-charge force and the coherent synchrotron radiation force for short electron bunches in circular motion using the retarded Green's function technique

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The space-charge forces for short electron bunches in circular motion can be very different from the space-charge forces for short electron bunches undergoing straight-line motion. The two major effects introduced by the circular motion are an off-axis, so-called ''noninertial space-charge'' effect, in which there is essentially no net energy loss of the bunch, and a coherent synchrotron radiation effect, in which the bunch radiates coherent energy. The consequence of these effects is a potentially large growth in the electron bunch's transverse emittance. We derive an expression for these forces from a Green's function approach, starting with the definitions of the retarded scalar and vector potentials. In particular, we find an expression for the total electric field along the direction of motion from a short line of charge in circular motion. These expressions in turn can be used in numerical particle simulations to estimate the amount of emittance growth, including the effects of suppressing the coherent synchrotron radiation by reducing the beam pipe dimensions. [S1063-651X(96)10607-3]

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I. INTRODUCTION

The space-charge forces on an electron bunch are a major consideration in accelerator design. Forces along the direction of motion can lead to a significant energy redistribution of the particles, which, in turn, can lead to a large growth in the bunch's transverse emittance. For an electron bunch that is not being accelerated, these forces scale inversely with the square of the relativistic mass factor γ. Because of this fact, space-charge forces are often ignored in accelerator designs where the beam is at high energy (greater than 1 GeV). However, recent work [1–3] has identified two space-charge-induced forces for beams in circular motion that are mostly independent of energy. These forces can cause a significant emittance growth, even at high energy. The first of these effects is considered a space-charge curvature effect and is known as the noninertial space-charge force, in which the energies of the particles are modified with little total loss by radiation. The second effect is known as the coherent synchrotron radiation force, in which the bunch radiates coherently. The coherent synchrotron radiation depends on the beam energy only through the normalized beam velocity β, and does not diverge for large beam energies as does the single-particle synchrotron radiation. For sufficiently large bunch charges and short bunch lengths, the coherently radiated energy will be much larger than the incoherently radiated energy.

Both the noninertial space-charge force and the force from the coherent synchrotron radiation will lead to a redistribution of the energy of those particles that are in circular motion within an achromatic bend in a high-brightness accelerator. This redistribution in turn can lead to an unacceptable increase in the beam's emittance that would be roughly independent of beam energy. Previously, the analytic technique used to calculate the noninertial space-charge effect (perturbation expansion of the wave equation for the vector and scalar potentials) does not easily include the effect of radiation. Conversely, the technique used to find the radiation effect (explicit calculation of the retarded fields along the bunch's trajectory) does not lead to off-axis effects (such as the noninertial space-charge force). In addition, the earlier techniques cannot easily be used to quantify the emittance growth in typical accelerator bend systems. In this paper, we use the retarded Green's function for the scalar and vector potentials to establish a formalism that allows us to numerically evaluate both the noninertial space-charge effect and the radiation effect everywhere. We will calculate the noninertial space-charge and radiation effects for a line of charge (to eliminate the additional single-particle radiation force); we can construct the forces for arbitrary distributions simply by superimposing the solutions for many such lines, displaced both longitudinally and transversely. These solutions can be easily incorporated in particle-tracking simulation codes, where line-by-point space charge calculations are common [4]. Using this technique in a suitably modified simulation code, we calculate the transverse emittance growth of the beam as it passes through an achromatic bend. We present the emittance growth scalings with respect to energy, bunch length, transverse bunch size, bend radius, and bend angle, and explore the effect of suppressing the coherent synchrotron radiation by reducing the beam pipe size.

II. CALCULATION OF THE TOTAL SPACE-CHARGE FORCE FROM A LINE OF CHARGE

In this section, we will find an expression for the total space-charge force from a line of charge in circular motion that (1) includes both the noninertial space-charge force and the coherent synchrotron force, and (2) can be included in a numerical particle-tracking code.

Consider the case of a short line of charge traveling in a circle, where we define the θ direction to be along the direction of motion, and where we use cylindrical coordinates.

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The electric field along the direction of motion is given by

$$E_\theta = -\frac{\partial A_\theta}{\partial t} - \frac{1}{r}\frac{\partial \phi}{\partial \theta}, \tag{1}$$

where A_θ is the azimuthal component of the vector potential and ϕ is the scalar potential. If the vector potential $\vec{A}$ is written as $\vec{A}=(\delta A_r,(\beta/c)\phi+\delta A_\theta,0)$, it was shown in Ref. [1] that

$$E_\theta = E_{\text{usual}}\left(1-\gamma^2\beta^2\frac{x}{R}\right)+\sum_n jn\omega\,\delta A_{\theta,n} \tag{2}$$

for the line of charge, where $\delta A_{\theta,n}$ is the deviation of the azimuthal component of the vector potential from the normalized scalar potential for the nth harmonic, R is the radius of curvature of the beam's circular motion, x is the transverse displacement of the observer location from the line's trajectory (in the bending plane), $\omega=\beta c/R$, and E_{usual} is the component of the force that scales inversely proportional to γ^2. The second term represents the contribution from the noninertial space-charge effect, and the third term leads to the coherent synchrotron radiation. It has been shown [1] that the noninertial space-charge term results from the curvature of the charge and current boundary conditions of the beam, and the radiation term results from an outgoing radiation boundary condition for large radii. In principle, $\delta A_{\theta,n}$ can be found by solution of the wave equation for the vector potential, but this is hard to do because the solution for $\delta A_{\theta,n}$ must be extended far away from the line of charge in order to establish the outgoing boundary condition.

Another approach to find E_θ induced from a line of uniform charge of length δ is to start with the scalar potential and the azimuthal component of the vector potential using the retarded potential formalism given in [5]

$$\phi = -\frac{R}{4\pi\varepsilon}\int_{\zeta_r}^{\zeta_f}\frac{\lambda}{r_{\text{ret}}-\vec{r}_{\text{ret}}\cdot\vec{u}_{\text{ret}}/c}\,d\zeta \tag{3}$$

and

$$A_\theta = -\frac{R}{4\pi\varepsilon c}\int_{\zeta_r}^{\zeta_f}\frac{\lambda\beta\cos\zeta'}{r_{\text{ret}}-\vec{r}_{\text{ret}}\cdot\vec{u}_{\text{ret}}/c}\,d\zeta, \tag{4}$$

where the variable ζ is the azimuthal angle relative to the observer location, the path of the integral is over the line of charge (at the present time), $\lambda=Q/\delta$ is the current density, $\vec{r}_{\text{ret}}$ is the vector from the source point (at position ζ) at the retarded time to the observer location, $r_{\text{ret}}=|\vec{r}_{\text{ret}}|$, $\vec{u}_{\text{ret}}$ is the retarded velocity of that point at the retarded time, ζ' is the retarded angle of the point's velocity, all shown in Fig. 1, and ζ_f and ζ_r are the present azimuthal angles of the front and rear of the bunch, respectively. Note that ζ' and ζ are considered positive if they lie behind the observer position. Recall that x is the transverse displacement of the observer point from the circular trajectory in the plane defined by the trajectory. We can also define a transverse displacement y of the observer point out of the plane of the trajectory, and a total transverse displacement $\rho=\sqrt{x^2+y^2}$.

These equations for the scalar and vector potentials are hard to solve, but fortunately we will not have to. Equation (1) demonstrates that we only have to know the derivatives of the potentials to find the electric field in the direction of motion. Note that the integrands for both the scalar and the azimuthal components of the vector potentials are only functions of the separation of the source and observer locations, and the radius of curvature, and not of the absolute azimuthal position or time as long as the path of the integral is defined relative to the source particle. Thus the derivatives indicated in Eq. (1) are trivial and are equal to the integrand evaluated at the limits of integration times the respective derivatives of the limits themselves.

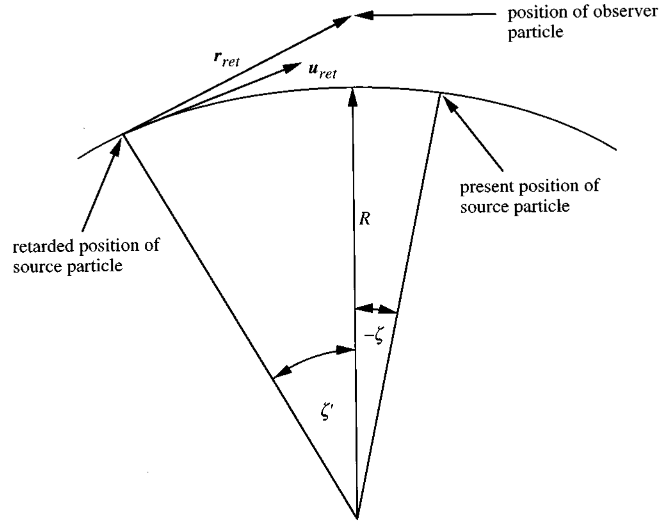

FIG. 1. Geometry defining the observer position, the present angle of the source particle ζ, and the retarded angle of the source particle ζ' for circular motion.

Now note that the position of the front of the bunch relative to an observer point at an azimuthal angle θ is given by $\zeta_f=\zeta_0-\beta ct/R+\theta$, for some ζ_0, and the position of the rear of the bunch is given by $\zeta_r=\zeta_0-\beta ct/R+\theta+\delta/R$. The differentiation in Eq. (1) for the azimuthal electric field results in (to lowest order in x/R)

$$E_\theta=\frac{\lambda}{4\pi\varepsilon}\frac{1}{r_{\text{ret}}-\vec{r}_{\text{ret}}\cdot\vec{u}_{\text{ret}}/c}\left(\frac{1}{\gamma^2}-\beta^2\frac{x}{R}+\beta^2\,[1-\cos(\zeta')]\right)\Bigg|_{\zeta_r}^{\zeta_f}. \tag{5}$$

Each of the three terms in parantheses is easily identified with a physical mechanism: the first term is the usual space-charge term, and vanishes if the beam is ultrarelativistic; the second term is the noninertial space-charge term; and the third term is the coherent synchrotron radiation term. It is clear that the second and third terms vanish if the radius of curvature of the circular motion becomes infinitely large; thus the noninertial space-charge force and the coherent synchrotron radiation force exist only when the beam is in circular motion.

In order to evaluate the effects of the noninertial space-charge force and the coherent synchrotron radiation force,

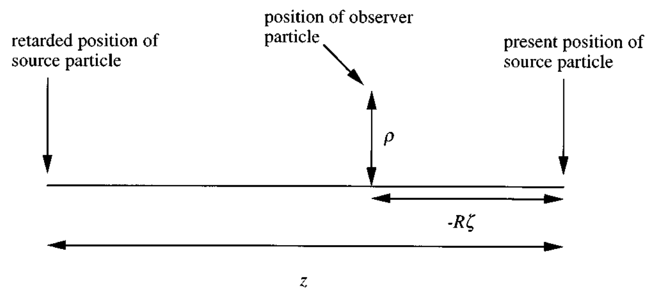

FIG. 2. Geometry defining the observer position, the present position of the source particle $R\zeta$, and the retarded position of the source particle $R\zeta+z$ for straight-line motion.

we need to find the retarded angle and evaluate the denominator in the second fraction. The retarded angle ζ' satisfies the transcendental equation

$$\beta^2R^2(\zeta'-\zeta)^2=\rho^2+2R(R+x)(1-\cos\zeta'), \quad (6)$$

which is, in general, hard to solve. Later, we will find solutions in certain limits that yield interesting results. Note that the retarded times depend strongly on the radius of curvature. What is referred to above as the ''usual'' space-charge term is not, in general, the same as the space-charge term that a bunch in straight-line motion would experience; only the scaling with beam energy is the same.

Using the geometry in Fig. 1, we find that the denominator of the second fraction in Eq. (5) is given by

$$r_{\rm ret}-\vec{r}_{\rm ret}\cdot\vec{u}_{\rm ret}/c=\sqrt{2R(R+x)(1-\cos\zeta')+\rho^2}-\beta R\ \sin\zeta' -\beta x(1-\cos\zeta')\sin\zeta'. \quad (7)$$

If the retarded angle is very small (for example, if the retarded distance is very small or the radius of curvature is very large), the denominator is $\sqrt{\zeta^2R^2+\rho^2/\gamma^2}$, where ζ is the unretarded position of that end of the bunch [5].

III. RELATIVELY STRAIGHT-LINE MOTION LIMIT

If $\gamma^2R\zeta$ is much less than the radius of curvature of the bunch's circular motion, for all positions ζ within the bunch, the motion between the retarded time and the present time is essentially straight line. Let us consider a very large radius of curvature relative to the dimensions of the beam, so that the motion is essentially straight over the dimensions of the beam. Now let us consider an observer point at a radius ρ from the line, and a source point at a position $R\zeta$ behind the observer point on the line, shown in Fig. 2. If the motion is straight, the retarded position of the source point is given by $R\zeta+z$ where the retarded distance is

$$z=R\zeta\beta^2\gamma^2+\gamma^2\sqrt{R^2\zeta^2\beta^4+\beta^2(R^2\zeta^2+\rho^2)/\gamma^2}. \quad (8)$$

The retarded distance is a strong function of the separation $R\zeta$ between the source point and the observer point. For example, if the beam is highly relativistic, $z=2R\zeta\beta^2\gamma^2$ if the observer point is in front of the source point (ζ is positive), $z=-\beta^2(R^2\zeta^2+\rho^2)/2R\zeta$ if the observer point is behind the source point (ζ is negative), and $z=\beta\gamma\rho$ if they are at the same position ($\zeta=0$). The retarded position is small only if the source point is in front of the observer point, and the retarded position can become extremely large if it is not. If the radius of curvature of the circular motion is large compared to all retarded positions, the circular motion can be considered essentially linear.

In this limit, the retarded distance denominator in Eq. (7) becomes $\sqrt{(R\zeta)^2+\rho^2/\gamma^2}$, and the azimuthal electric field is given by

$$E_\theta=\frac{\lambda}{4\pi\varepsilon}\frac{1}{\sqrt{(R\zeta)^2+\rho^2/\gamma^2}}\left(\frac{1}{\gamma^2}-\beta^2\frac{x}{R}+\beta^2 \times[1-\cos(\zeta-z)]\right)\Bigg|_{\zeta_r}^{\zeta_f}. \quad (9)$$

The noninertial space-charge force is the same as the space-charge force for straight-line motion, multiplied by γ^2x/R, and leads to no net energy loss of the bunch. Its characteristic magnitude (ignoring the $Q/4\pi\varepsilon$ factor) is $a/R\delta^2$, where a is the bunch radius and δ is the bunch length. The coherent synchrotron radiation force term is only non-negligible for source particles behind the observer point (where the retarded position can become large), and its characteristic magnitude for those particles is $2(\gamma^2-1)^2/R^2$. Because both $\gamma^4/R^2\ll1/\delta^2$ (the straight-line assumption above), and $a/R\ll1$, either term could dominate; however, if the radius of curvature is increased (keeping everything else constant), the coherent synchrotron radiation effect integrated over a given bending angle vanishes, whereas the effect from the noninertial space-charge force remains a constant.

IV. ULTRARELATIVISTIC MOTION

In this limit, we assume β equals unity. This is a much more interesting limit, and leads to coherent synchrotron radiation that actually becomes larger (integrated over a given bending angle) as we increase the radius of curvature of the circular motion.

There are two regimes within this limit where we can solve Eqs. (5) and (6): (i) a pencil beam with no transverse extent, and (ii) a pancake beam with no longitudinal extent.

(i) Pencil beam. Now we are assuming that the beam has no transverse size ($a\sim0$). The transcendental expression for the retarded position to fourth order in the retarded angle is

$$\zeta^2-2\zeta\zeta'=-\zeta'^4/12, \quad (10)$$

which has the solution $\zeta'\cong\zeta/2$ for source particles in front of the observer point (ζ negative), and $\zeta'\cong(24\zeta)^{1/3}$ for source particles behind the observer point (ζ positive). By consideration of the other terms in Eq. (6), we see that this limit is valid if $a/R\ll(24\zeta)^{2/3}/6$, which can always be made valid for a large enough radius of curvature (recall that $\zeta\sim\delta/R$). This limit is valid for most of a 1-ps-long, 1-mm-radius bunch in a bend with a radius of curvature greater than 20 cm.

The particles in front of the observer point lead to small contributions of $a/R\delta^2$ for the noninertial space-charge force and $1/R^2$ for the coherent synchrotron radiation force. Likewise, the noninertial space-charge force from source particles behind the observer point is about $a/3R\delta^2$. However, the force from the coherent synchrotron radiation due to the particles behind the observer point scales as

$$\frac{1-\cos\zeta'}{\delta R(\zeta'-\sin\zeta'-\zeta)} \cong \frac{\zeta'^2/2}{\delta R(\zeta'^3/6-\zeta)} = \frac{2}{3^{1/3}\delta R \zeta^{1/3}} = \frac{2}{3^{1/3}\delta^{4/3}R^{2/3}}, \tag{11}$$

which in fact becomes larger if the radius of curvature is increased, keeping all other parameters fixed. This dependence on the radius is the same as reported in Refs. [3] and [6].

(ii) Pancake beam. In this limit, we assume that δ is very small. From Eq. (6) this means that the bunch length obeys $\delta \ll a^2/R\zeta'$. Keeping the expression for the retarded angle to fourth-order again, we now find that

$$\delta'^2 = \frac{6}{R}\,(x+\sqrt{x^2+\rho^2/3}), \tag{12}$$

for both x positive and negative. The requirement on the bunch length for this limit now becomes $\delta \ll a^{3/2}/R^{1/2}$. This limit is approached for bunch lengths less than 100 fs for a bunch radius of 1 mm and a bend with a radius of curvature of 1 m. For longer bunch lengths or larger radii of curvature, a smaller and smaller fraction of the beam is within this limit. For this case, the denominator in Eq. (5) becomes

$$R\zeta' - (R+x)\sin\,\zeta' \cong \zeta'\,\sqrt{x^2+\rho^2/3}. \tag{13}$$

Using this, we can show that the noninertial space-charge force scales as

$$\frac{a/R}{\delta\zeta'\,\sqrt{x^2+\rho^2/3}} \approx \frac{1}{6^{1/2}\delta R^{1/2}a^{1/2}}, \tag{14}$$

and the force from the coherent synchrotron radiation scales as

$$\frac{1-\cos\zeta'}{\delta\zeta'\,\sqrt{x^2+\rho^2/3}} \cong \frac{\zeta'^2/2}{\delta\zeta'\,\sqrt{x^2+\rho^2/3}} \approx \frac{3^{1/2}}{2^{1/2}\delta R^{1/2}a^{1/2}}. \tag{15}$$

Thus, in this limit, both forces have a similar magnitude, and scaling, and both can lead to a very significant degradation of the beam quality for large radii of curvature. However, as the beam radius is increased, the amount of the bunch that contributes to this force scales as $1/R^{1/2}$, and the forces integrated over the bunch distribution do not grow unbounded.

V. NUMERICAL SIMULATIONS USING THE NONINERTIAL SPACE-CHARGE AND COHERENT SYNCHROTRON RADIATION FORCES

In this section, we will incorporate the expression found for the total space-charge force from a line of charge in circular motion into a numerical particle-tracking code. This will be used to quantify the emittance growth due to the noninertial space-charge force and the coherent synchrotron force for a typical accelerator bend system, over a wide range of bunch and bend parameters. The emittance growth calculated for typical parameters is large enough to exceed the maximum allowed emittance for many proposed applications.

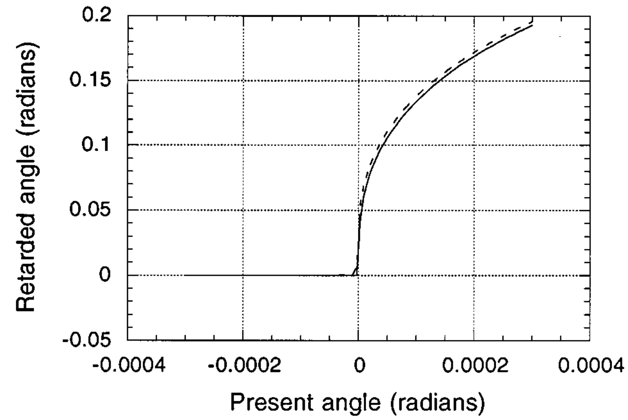

FIG. 3. Comparison of the predicted retarded angle (solid line) using the approximations after Eq. (10) and the actual retarded angle (dashed line) found by solving Eq. (6).

It is straightforward to numerically evaluate the space-charge forces using the expressions in Eqs. (5) and (6) for either a single line of charge or for a bunch assembled from a collection of such lines. The main issue in a simulation of this sort is the calculation of the retarded angle relative to the observer point (see Fig. 1). It is tempting to use the ultrarelativistic, pencil-beam approximations [given in the paragraph after Eq. (10)] for the retarded angle, because in that case, the retarded angle is a very simple function of the present angle ζ. In Fig. 3 we plot the retarded angle calculated in this manner (the solid line) and the actual retarded angle calculated by direct solution of Eq. (6) (the dashed line), as a function of present azimuthal angle ζ between the source and observer points, with $x=1$ mm, $y=0$ mm, a radius of curvature of 1 m, and a beam energy of 100 MeV. (Recall that a positive angle means that the observer point is in front of the source particle.) A 1-ps bunch extends over about 3×10^{-4} rad. The approximate retarded angle is very accurate, with some deviation near a present angle of 0 rad. Since the force from the coherent synchrotron radiation vanishes as the retarded angle goes to zero, we would expect that this approximation for the retarded angle is fine for this force. In Fig. 4 we plot the coherent synchrotron radiation force from one edge of a line of charge [by using one of the limits in Eq. (5)] using the approximation for the retarded angle (the solid line) and using the actual retarded angle [numerically solving Eq. (6)] as a function of the present angle ζ. In this and the following figure, Fig. 5, the transverse offsets, the radius of curvature, and the beam energy are the same as in the preceding figure, Fig. 3. There is no significant deviation, and this approximation can predict the effect from the coherent synchrotron radiation force to within 10%. However, the noninertial space-charge force is only significant if the present angle is very close to zero, and we would expect that small errors in the approximation of the retarded angle can become important. Note that the retarded angle enters into this force only in the denominator, as described in Eq. (7). In Fig. 5 we plot the noninertial space-charge force using the approximation for the retarded angle (the solid line) and using the actual retarded angle (the dashed line). For this force,

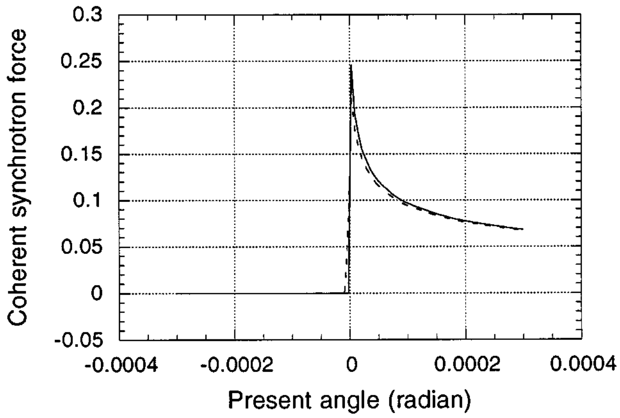

FIG. 4. Comparison of the approximate coherent synchrotron radiation force (solid line) using the approximation for the retarded angle and the actual coherent synchrotron radiation force (dashed line).

the actual retarded angle must be used in numerical simulations.

The retarded angle can be found by recursively solving Eq. (6). However, because the retarded angle must be found separately for each set of points in the simulation for each time step, the most efficient iteration scheme must be used. An extremely efficient scheme is defined by these steps: (1) use the pencil-beam approximations after Eq. (10) for a first guess of the retarded angle ζ'^*, (2) use

$$\zeta' = \frac{\sqrt{2R(R+x)(1-\cos\zeta'^*)+\rho^2}}{\beta R} + \zeta \tag{16}$$

[from Eq. (6)] to make a refined first guess for the retarded angle, (3) solve Eq. (6) for the present angle from the guess for the retarded angle and for a slightly shifted retarded angle, and use a linear extrapolation to make a new guess for the retarded angle that will lead to the desired present angle. Step (3) can be iterated until the relative error between the actual present angle and the present angle corresponding to the guess for the retarded angle is sufficiently small. For

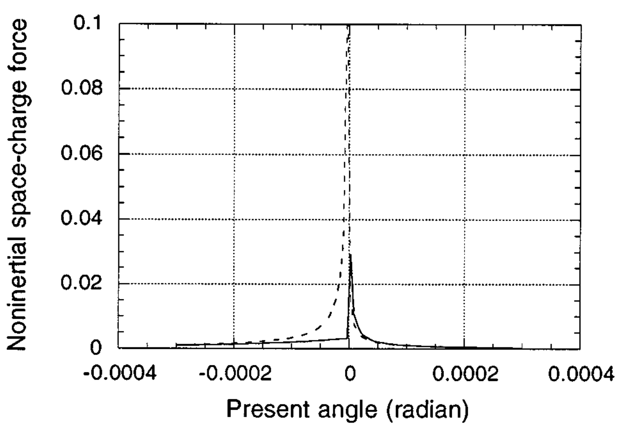

FIG. 5. Comparison of the approximate noninertial space-charge force (solid line) using the approximation for the retarded angle and the actual noninertial space-charge force (dashed line).

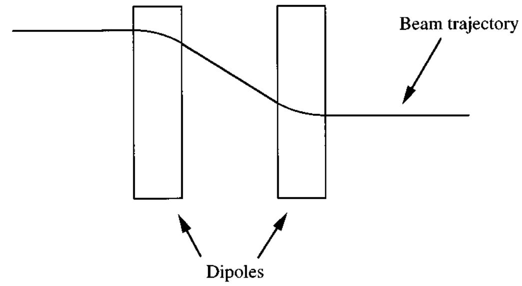

FIG. 6. Two-dipole ''dogleg'' geometry used for the numerical emittance growth calculations.

relative errors of 1% or less, typically only 5–10 iterations are needed, even with present angles very close to zero.

Using an existing line-to-point space-charge routine in the particle-tracking simulation code PARMELA [7], we have included the forces in Eq. (5) to determine the effect of the noninertial space-charge and the coherent synchrotron radiation forces for a short beam, a two-dipole ''dogleg,'' shown in Fig. 6. We will quantify the amount of the beam quality degradation caused by these effects by calculating the transverse normalized rms emittance growth in this system, where we define the transverse normalized rms emittance by

$$\varepsilon = \beta\gamma\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}, \tag{17}$$

and where x' is the angular divergence of a particle relative to the centroid of the bunch and the brackets indicate ensemble averages over the entire beam distribution. Note that there is no emittance growth in the absence of space-charge forces for an initially monoenergetic beam in the two-dipole system shown in Fig. 6, if the dipoles are identical. For these simulations, the beam path in the dipoles was assumed to be 10 cm and the dipoles are separated by 1.8 cm. In addition, the bunch was nominally assumed to contain 1 nC of charge and to have a Gaussian longitudinal distribution with a full width at half maximum (FWHM) of 1 ps, to have a uniform radial distribution with a 1-mm radius, and to be at 400 MeV; and the bend angle was assumed to be 5° (the radius of curvature of the circular motion was then about 115 cm). The initial beam divergence (and consequently the initial emittance) was assumed to be zero. The emittance growth from the ''normal'' space-charge force in Eq. (5) (proportional to $1/\gamma^2$) was about 0.02 mm mrad for these parameters, for comparison to the following results.

PARMELA was modified to include separately both the noninertial space-charge force and the force from the coherent synchrotron radiation, in order to clearly distinguish the effects from each force. In the next seven figures, the emittance growth from each force will be presented separately, with the solid line for the noninertial space-charge force and the dashed line for the force from the coherent synchrotron radiation. By including both effects in a simulation, we have verified that the overall emittance growth is roughly the sum of the individual emittance growths in quadrature. In Fig. 7 we plot the emittance growths from these effects as a function of the number of particles included in the simulation. The emittance growths do not depend strongly on the number of particles, which was expected for the coherent synchrotron force but not necessarily for the noninertial space-

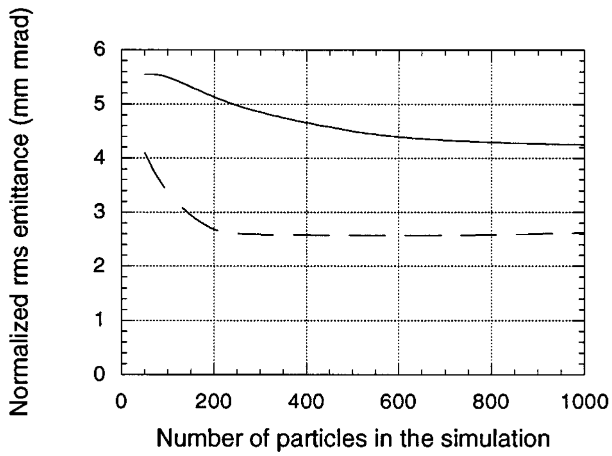

FIG. 7. Emittance growth for a 1-ps, 1-kA, 1-mm-radius, 400-MeV bunch in the 5°, two-dipole system as a function of the number of particles used in the simulations. The emittance growth from the noninertial space-charge force is shown as a solid line and the emittance growth from the coherent synchrotron radiation force is shown as a dashed line.

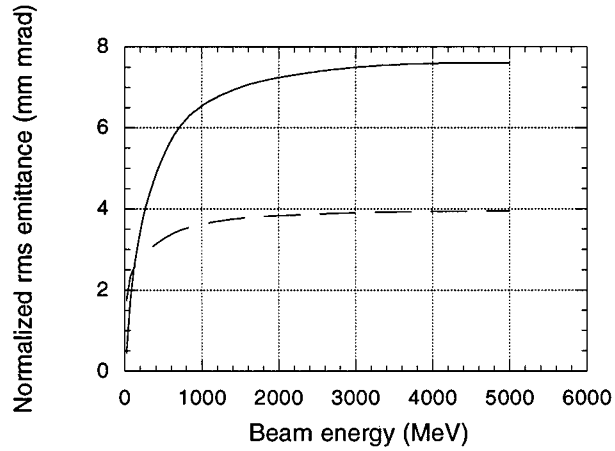

FIG. 8. Emittance growth for a 1-ps, 1-kA, 1-mm-radius bunch in the 5° two-dipole system as a function of beam energy. The emittance growth from the noninertial space-charge force is shown as a solid line and the emittance growth from the coherent synchrotron radiation force is shown as a dashed line.

charge force. For all the subsequent figures, 500 particles were used in the numerical calculations of the emittance growth from the coherent synchrotron radiation force; because the simulations of the noninertial space-charge force took roughly ten times more computer time (resulting from the iterations of the retarded times), we used only 200 particles for the numerical calculations of the emittance growth from the noninertial space-charge force. The error in the emittance growth from the noninertial space-charge force introduced by using so few particles was estimated by comparing the results of simulations of only a single time step with up to 4000 particles. The results with 200 particles overestimate the emittance growth by about 20–25 %. The error introduced by the transitions at the start and at the end of the dipoles is not significant because only particles very near the observer point contribute a large noninertial space-charge force. However, the transitions can influence the forces from the coherent synchrotron radiation significantly. In the simulation, all particles are assumed to be in circular motion if the center of the bunch is in the dipole. As a result, during the starting transition, some particles are considered to be in circular motion while they are actually undergoing straight-line motion (which leads to no noninertial space-charge force or coherent synchrotron radiation force). During the ending transition, these same particles are considered to have straight-line motion while they are actually still in circular motion. The actual coherent synchrotron radiation force induced by these particles on the rest of the bunch during the ending transition is somewhat less than the assumed coherent synchrotron radiation force during the starting transition because the retarded angle is underestimated. Considering the form of the coherent synchrotron radiation force in Fig. 4, we see that a larger retarded angle leads to a lower force, but also that the force is fairly flat. Estimates indicate that the coherent synchrotron radiation forces calculated in this manner are on the order of 25% too large for our nominal 1-ps bunch and a 5° bend-angle dogleg, which directly translates into an error of the same size for the emittance growth. It should be noted that the retarded angles are spread over a smaller fraction of the bend as the bend angle is increased, roughly to the $\frac{1}{3}$ power. Also, as the bunch length decreases or the radius of curvature of the bunch trajectory increases, this error will decrease, and will eventually vanish for a very short bunch in a bend with a very large radius of curvature. These errors should be kept in mind; despite them, the scalings seen in the following figures should be valid. Note that for the nominal case described above, the normalized rms emittance growths are nearly equal in size and about 5 mm mrad for the noninertial space-charge force and 3 mm mrad for the coherent synchrotron radiation force. Even with these modest parameters (a 1-ps, 1-kA, 1-mm-radius beam), these emittance growths are larger than the target emittances of many future, high-brightness accelerators, and these emittance growths must be avoided. For the nominal case, the coherent synchrotron radiation leads to a fractional energy loss of the total bunch energy of 2×10^{-4} and the noninertial space-charge force to a smaller fractional energy loss of 5×10^{-6}.

In Fig. 8, we plot the emittance growths from these effects as a function of beam energy, and see that both emittance growths approach a limit as the energy is increased. It is interesting to note that the emittance growths from these effects for energies below 50 MeV are about an order of magnitude smaller than their asymptotic values.

In Fig. 9 we plot the emittance growth as a function of the bunch radius. Earlier estimates of the noninertial space-charge force [1] predicted an emittance growth that scales as the square of the bunch radius, which appears to be only true for bunch radii <1 mm.

In Fig. 10 we plot the emittance growth as a function of bunch length. Earlier estimates of the noninertial space-charge force [1] predicted an emittance growth that scales inversely as the square of the bunch length, which appears valid at the longer bunch lengths. Detailed analyses of the simulations indicate that the nonzero bunch radius leads to an appreciable bunch lengthening while the bunch is in the dipoles, and this could be influencing the results for very

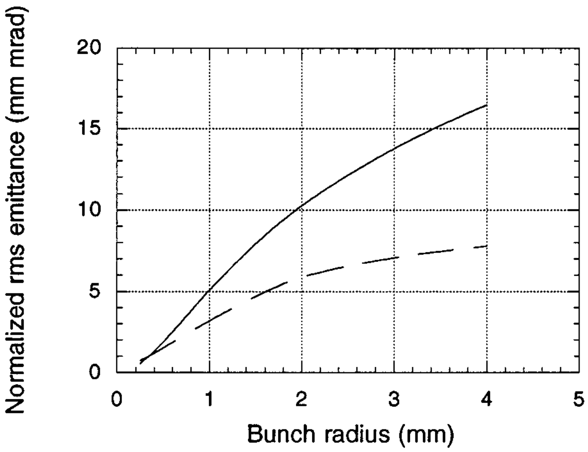

FIG. 9. Emittance growth for a 1-ps, 1-kA, 400-MeV bunch in the 5°, two-dipole system as a function of the bunch radius. The emittance growth from the noninertial space-charge force is shown as a solid line and the emittance growth from the coherent synchrotron radiation force is shown as a dashed line.

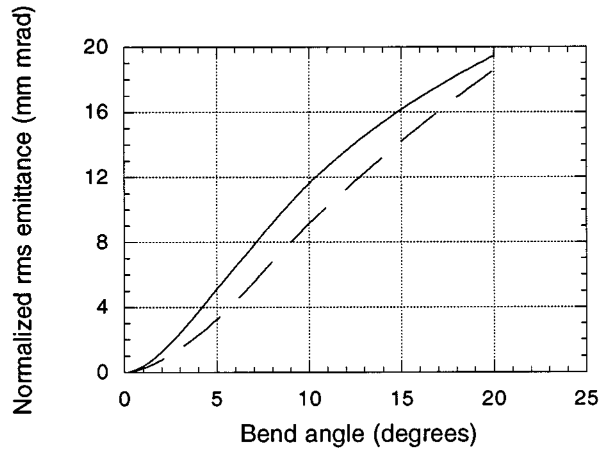

FIG. 11. Emittance growth for a 1-ps, 1-kA, 1-mm-radius, 400-MeV bunch in the two-dipole system as a function of the bend angle. The emittance growth from the noninertial space-charge force is shown as a solid line and the emittance growth from the coherent synchrotron radiation force is shown as a dashed line.

short bunch lengths. Recall that the error in the calculation of the coherent synchrotron radiation force increases as the bunch length is increased.

In Fig. 11 we plot the emittance growth as a function of bend angle. For very small bend angles, the emittance growth appears to scale as the square of the bend angle, as predicted before [1], but the growth becomes more linear for bend angles of 5° and beyond. Again, this is probably due to a distortion of the bunch shape as the bunch becomes curved in the dipoles. The bunch is distorted more at the higher bend angles.

In Fig. 12 we plot the emittance growth as a function of the bend radius of curvature, while maintaining 5° bends in the dipoles. As predicted, the emittance growth from the coherent synchrotron radiation does indeed increase with an increased bend radius (and roughly as the $\frac{1}{3}$ power), whereas the emittance growth from the noninertial space-charge force decreases. Note that the sum of the two emittance growths in quadrature leads to a total growth of about 6 mm mrad, roughly independent of bend radius over this range.

A very nice feature of this simulation technique is that it is very easy to include the effect of image charges in the beam box walls by including image lines of charge. In this manner, we can quantify the effect of suppressing the coherent synchrotron radiation by making the beam box dimensions such that the microwave radiation from this effect cannot propagate, as suggested in Refs. [2,3,6]. The major effect on the bunch is that the image charges from the beam box produce a counterforce on the observer particles in the bunch; however, even if the radiation is suppressed it is not clear that the forces leading to the emittance growth are suppressed. For simplicity, we will consider the case where

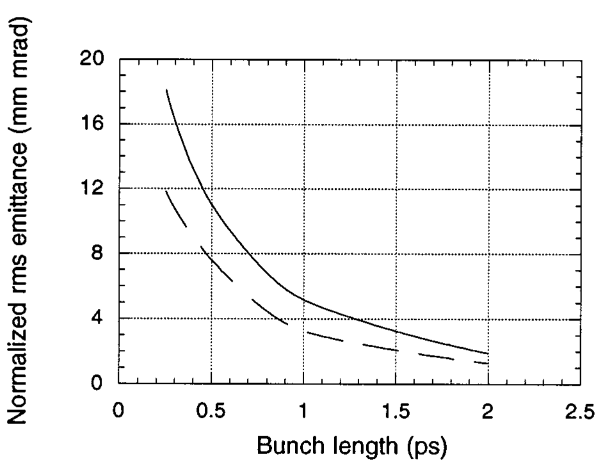

FIG. 10. Emittance growth for a 1-kA, 1-mm-radius, 400-MeV bunch in the 5°, two-dipole system as a function of the bunch length. The emittance growth from the noninertial space-charge force is shown as a solid line and the emittance growth from the coherent synchrotron radiation force is shown as a dashed line.

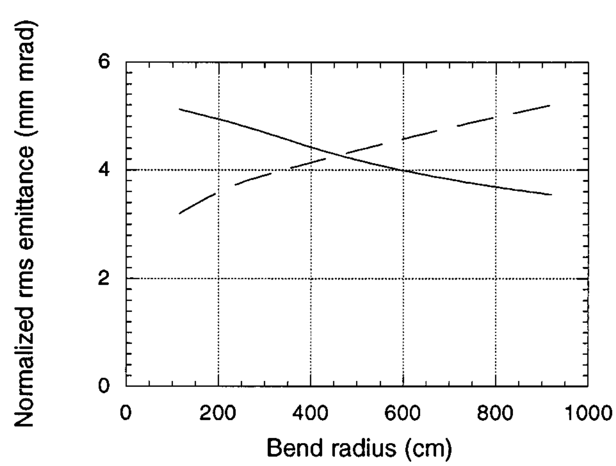

FIG. 12. Emittance growth for a 1-ps, 1-kA, 1-mm-radius, 400-MeV bunch in the 5° two-dipole system as a function of the bend radius. The emittance growth from the noninertial space-charge force is shown as a solid line and the emittance growth from the coherent synchrotron radiation force is shown as a dashed line.

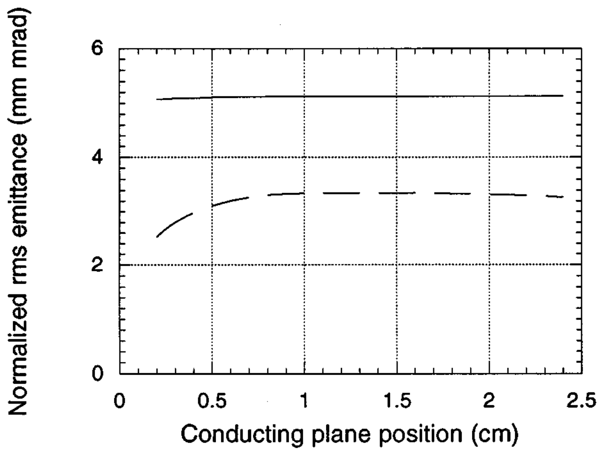

FIG. 13. Emittance growth for a 1-ps, 1-kA, 1-mm-radius, 400-MeV bunch in the 5°, two-dipole system as a function of the separation between the bunch center and a conducting plane in order to determine the effect of suppressing the coherent synchrotron radiation. The emittance growth from the noninertial space-charge force is shown as a solid line and the emittance growth from the coherent synchrotron radiation force is shown as a dashed line.

there is only a single conducting plane near the bunch, parallel to the bend trajectory (in this case there is only one image line of charge, instead of an infinite number as there would be with two planes). In the final figure, Fig. 13, we plot the emittance growth from these forces as a function of the separation between the bunch center and the conducting plane. There is essentially no change in the emittance growth from the noninertial space-charge force, and only a modest decrease in the emittance growth from the coherent synchrotron radiation force. We can observe that this suppression technique would only be useful for (1) very large radius bends where the emittance growth from the noninertial space-charge force vanishes and (2) very small radius bunches (<0.1 mm) where the bunch can be placed very close to the beam box wall (well within 1 mm for bunch lengths <1 ps).

VI. CONCLUSION

We have developed a formalism that describes the noninertial space-charge force and the force from the coherent synchrotron radiation. This formalism can be used in numerical simulations of accelerator-bending systems in order to determine the emittance growth of a high-brightness beam in a bend. For the nominal bunch-dipole system we considered, the emittance growth is very significant, and roughly scales as theoretically predicted. Over the range of the numerical simulations, the emittance growth from both effects together is roughly independent of the radius of curvature of the bunch in the dipoles. In addition, very little effect on the emittance growth was observed by suppressing the coherent synchrotron radiation. These effects may be very important for future, high-brightness accelerators with very short bunch lengths.

ACKNOWLEDGMENT

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[1] B. E. Carlsten and T. O. Raubenheimer, Phys. Rev. E **51**, 1453 (1995).
[2] J. B. Murphy, S. Krinsky, and R. L. Gluckstern (unpublished).
[3] Ya. S. Derbenev, J. Rossbach, E. L. Saldin, and V. D. Shiltsev (unpublished).
[4] B. E. Carlsten, L. M. Young, M. J. Browman, H. Takeda, D. M. Feldman, P. G. O'Shea, and A. H. Lumpkin, Nucl. Instrum. Methods A **304**, 587 (1991).
[5] W. K. H. Panofsky and M. Philips, *Classical Electricity and Magnetism* (Addison-Wesley, Reading, MA, 1962), p. 343.
[6] J. S. Nodvick and D. S. Saxon, Phys. Rev. **96**, 180 (1954).
[7] L. Young (private communication).
[8] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), p. 392.